Research on atomic time algorithm of Cs fountain NTSC-F2 and masers

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Abstract*—The 'Paper time scale' calculated by multiple atomic clocks is an important reference for the generation of high-precision time signals, so improving the performance of the time scale is of great significance for timing system. Cesium fountain NTSC-F2 has high precision, and it is used by BIPM as one of the references for the calibration of EAL. Hydrogen maser clocks have good short-term frequency stability, which are an important part of the clock ensemble for NTSC timing keeping system. Combining the advantages of Cesium atomic fountain and Hydrogen maser clocks to calculate a combined time scale can effectively improve the performance of the time scale. The Vondrak-Cepek method is a combined smoothing algorithm that uses two independent data sequences to obtain the advantages of both. Based on Vondrak-Cepek method, the combined time scale algorithm of the cesium fountain and the hydrogen masers is proposed in this paper. Firstly, the cesium fountain NTSC-F2 was preprocessed and the hydrogen masers time scale was calculated. Then, the combined time scale based on the cesium fountain and masers time scale is calculated by the Vondrak-Cepek algorithm. The results show that the short-term stability of the combined time scale is better than that of the cesium fountain, Allan deviation of the combined time scale is 1.33E-15 at 3600s, which is 76% higher than that of the cesium fountain. Meanwhile, the long-term stability of the combined time scale is better than that of masers time scale, Allan deviation of the combined time scale is 2.45E-16 at 10 days, which is 49% higher than that of the masers time scale. The root mean square (RMS) of combined time scale relative to UTC is 0.14ns, which is 86% higher than that of the masers time scale, and 6% higher than that of the cesium fountain.

Keywords—Time scale, Cs fountain, hydrogen maser, Vondrak-Cepek smoothing method.

I. INTRODUCTION

The international time reference Coordinated Universal Time (UTC) is the most stable time scale. But UTC is only available with a latency that can reach 40 days. For the safety of the time application, the independent, stable and accurate local time scale play an important role in generating a physical signal, available in real time.

The cesium fountain serves as a primary frequency reference for reproducing the definition of "second", and it almost has no drift, so it has good long-term stability. Commercial hydrogen masers have good short-term stability, and it can operate continuously. Therefore, combining the advantages of cesium fountain and hydrogen masers, a combined time scale can be generated.

Vondrak-Cepek is a more general method of smoothing in which the estimation is done from two available independent series. Both series are combined to yield two smooth curves tied by the constraints assuring that he latter is the time derivative of the former. The first one curves fits well to the first series and the second one fits well to the second series. The goal is to make use of advantages of both series (Such as, long-term stability of the former and short-term stability of the latter) in one solution.

Firstly, the masers time scale was calculated with the ALGOS algorithm. Then the Vondrak-Cepek smoothing algorithm was used in combining the cesium fountain and the masers time scale. On the one hand, the fidelity of the combined time scale is close to the cesium fountain, and on the other hand, the smoothness is close to the masers time scale. Therefore, the stability of the combined time scale was improved.

II. THEORY

Vondrak-Cepek smoothing is an improvement on initial Vondrak smoothing. We can use Vondrak-Cepek smoothing when not only the values of the function itself but also its first time derivatives are measured. The method consists in finding a weighted compromise among three different conditions: smoothness of the searched curve, its fidelity to the observed function values and its fidelity to the observed first time derivatives. Its objective is to remove the high-frequency noise present in the observations of a time function and its first derivatives.

Two time series of observations are available – one with measured function values whose analytic expression is unknown (Series 1) and the other with measured time derivatives (Series 2). Both series are given at unequally spaced epochs that need not be necessarily identical, and the individual observations are given with different precision.

- Input series 1 be given as the observed function values$y_j{}'$,at instants $x_j$with weights p_j,$j = 1,2,\cdots n$
- Input series 2 be given as the observed values of first derivatives $\bar{y}_k{}'$, at instants $\bar{x}_k$with weights $\bar{p}_k$,$k = 1,2,\cdots \bar{n}$
- Output series of the searched function values on the smoothed curve be y_i at the instants x_i containing all values of x_j and$\bar{x}_k$, $i = 1,2,\cdots N$,$N \leq n + \bar{n}$.
- We define three quantities:

1) Smoothness of the curve

$$S = \frac{1}{x_N - x_1} \int_{x_1}^{x_N} \varphi'''^2(x)dx \tag{1}$$

Refer to (1), analytical expression of the function $\varphi(x)$is unknown so the value of its third derivative $\varphi(x)$ must be estimated numerically from the smoothed data y. The smoothed curve in the interval between two points $[x_{i+1}, y_{i+1}]$ and $[x_{i+2}, y_{i+2}]$ is defined as a third-order Lagrange polynomial $L_i(x)$ running through the four adjacent points. So, equation (1) is turn to (2).

$$S = \frac{1}{x_N - x_1} \sum_{i=1}^{N-3} \int_{x_{i+1}}^{x_{i+2}} L_i'''^2(x)dx \tag{2}$$

2) Fidelity of the smoothed curve to the observed values

$$F = \frac{1}{n} \sum_{i=1}^{N} p_i (y_i' - y_i)^2 \tag{3}$$

3) Fidelity of the smoothed curve to the observed first derivatives

$$\overline{F} = \frac{1}{\overline{n}} \sum_{i=1}^{N} \overline{p}_i (\overline{y}_i' - \overline{y}_i)^2 \tag{4}$$

In which the smoothed values of first derivatives $\bar{y}_i$ can be expressed in terms of the smoothed function valuesy_i.

We are looking for the smoothed curve (x_i, y_i) as a compromise among three different conditions.

- The curve should be smooth.
- The smoothed values should be close to the observed values of the function.
- The first derivatives of the smoothed curve should be close to the observed values of first derivatives.
- The adjustment is then done by minimizing a combination of the constraints above.

$$\begin{aligned} &Q = S + \varepsilon F + \overline{\varepsilon}\overline{F} = \min \\ &\Rightarrow \frac{\partial Q}{\partial y_i} = 0 \qquad i = 1, 2, \cdots N \end{aligned} \tag{5}$$

In which, $\varepsilon \geq 0, \bar{\varepsilon} \geq 0$, the larger are the values, the larger weight we assign to the fidelity to the observed function values or their first derivatives, and the closer the "smoothed" values are to the observations. Equation (5) is leading to the system of N linear equations with unknowns y_i .Obtained unknowns y_i by solving equations, y_i is the value of smoothing.

The following describes how to select the smoothing coefficients. The new method of choosing coefficients of smoothing based on the original smoothing error method is designed. The basic idea consists in finding a compromise between the weight of observed values in the smoothed curve and the weight of the observed first derivatives in the smoothed curve. The coefficients of smoothing are computed with the following function.

$$\sigma_{\varepsilon,\overline{\varepsilon}} = \sqrt{\alpha \frac{\sum_{i=1}^{N} p_i (y_i - y_i')^2}{N-3} + (1-\alpha) \frac{\sum_{i=1}^{N} \overline{p}_i (\overline{y}_i - \overline{y}_i')^2}{N}}, \alpha \in [0,1] \tag{6}$$

α is an adaptive factor. It determines the weighted compromise between the observed values and the observed first derivatives. The factor α has been chosen according to actual needs. After selectingα, the $\sigma_{\varepsilon,\bar{\varepsilon}}(\varepsilon, \bar{\varepsilon})$ was calculated by using (6), based on different $\varepsilon$and$\bar{\varepsilon}$. ε As X axis, $\bar{\varepsilon}$ as Y axis, $\sigma_{\varepsilon,\bar{\varepsilon}}(\varepsilon, \bar{\varepsilon})$ as Z axis, the surface plot was obtained. The coefficients of smoothing were chosen by the minimum of the surface.

III. METHODS

The characteristic of Vondrak-Cepek is to make use of advantages of both independent series in one solution. In the two series, the latter is the first time derivatives of the former. Cesium fountain and masers time scale are two independent series. They have different sampling intervals. The advantage of cesium fountain is long-term stability. The advantage of masers time scale is short-term stability. So, we can use Vondrak-Cepek in combining the cesium fountain and the masers time scale. In order to make the latter is the first time derivatives of the former. We use the phase data of the cesium fountain as the former, the frequency data of the masers time scale as the latter.

The F in the Vondrak-Cepek algorithm is weighted average of quadratic sum of difference between cesium fountain and the combined time scale. The $\bar{F}$ in the Vondrak-Cepek algorithm is weighted average of quadratic sum of difference between the first time derivatives of masers time scale and the first time derivatives of the combined time scale. The S in the Vondrak-Cepek algorithm is the requirement for the sum square of the third derivative of the curve. If the third derivative of the curve is small, the curve is very smooth.

From the principle of Vondrak-Cepek, it consists in finding a weighted compromise among smoothness of the searched curve, its fidelity to the cesium fountain and its fidelity to the masers time scale first time derivatives. The coefficients of smoothing can determine the weighted compromise. By selecting the coefficients of smoothing, make the long-term stability of the combined time scale is close to the cesium fountain, and the short-term stability is close to the masers time scale. Flow chart of the time scale combined algorithm based on Vondrak-Cepek was given in the following figure.

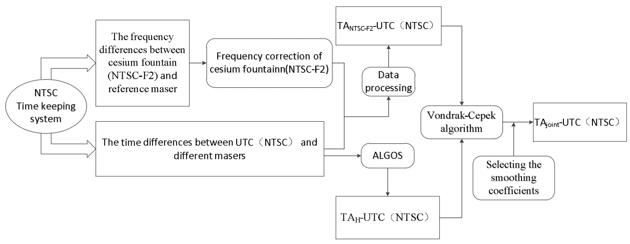

Fig.1. The time scale combined algorithm procedure

A. Cesium fountain data preprocessing and masers time scale calculation

The cesium fountain clock (No. NTSC-F2) and hydrogen masers in the NTSC time keeping system were used to calculate the time scale of the masers and the cesium fountain, respectively.

Using the measured data of the cesium fountain NTSC-F2 of MJD 60342~60399 (where MJD 60369~60373 is the dead time of cesium fountain NTSC-F2), after frequency shift correction and data preprocessing of NTSC-F2, the time difference data between the cesium fountain NTSC-F2 and UTC (NTSC) were calculated, which was denoted as $TA_{NTSC\text{-}F2}$-UTC (NTSC).

The same MJD 60342~60399, eight hydrogen masers in the NTSC time keeping system were selected. The ALGOS algorithm is used to calculate the masers time scale. The time difference data between the masers time scale and UTC (NTSC) was obtained, which was denoted as TA_H-UTC (NTSC).

The accuracy of the two time scales was verified according to the UTC-UTC (NTSC) published in the Circular T 434~435, and the results are shown in Figure 2. The stability of $TA_{NTSC\text{-}F2}$ and TA_H was calculated separately, and the results are shown in Figure 3.

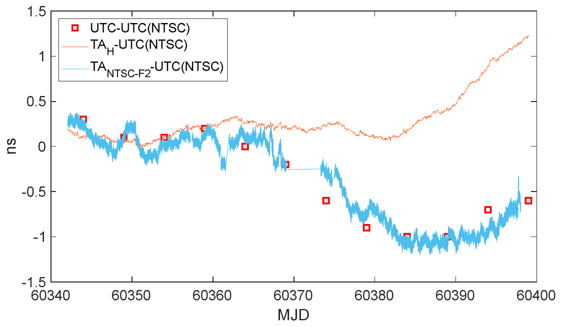

Fig.2. The time differences between UTC, TA_H, $TA_{NTSC\text{-}F2}$ and UTC(NTSC)

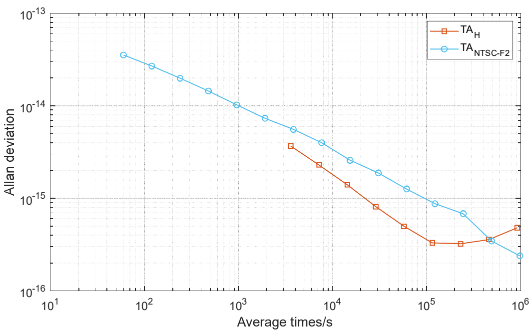

Fig.3. The frequency instability of TA_H and $TA_{NTSC\text{-}F2}$

Fig. 2 and Fig. 3 show that the short-term stability of the masers time scale is better than that of the cesium fountain, and the accuracy relative to UTC and long-term stability of the cesium fountain are better than those of the masers time scale.

B. Selecting the smoothing coefficients ε and $\bar{\varepsilon}$

Smoothing coefficients was calculated by using (6). The coefficients α determine the weighted compromise among the searched curve's fidelity to the cesium fountain and its fidelity to the masers time scale. The accuracy of cesium fountain is better than the masers time scale. We hope cesium fountain gets higher weight in the combined time scale. But when the cesium fountain in the dead time, the masers time scale gets higher weight in the combined time scale. The coefficient α has been chosen according to actual needs. Then, based on different coefficients of smoothing, the root mean square error of smoothed values ($\sigma_{\varepsilon,\bar{\varepsilon}}(\varepsilon,\bar{\varepsilon})$) was calculated by using (6). ε as X axis, $\bar{\varepsilon}$ as Y axis, $\sigma_{\varepsilon,\bar{\varepsilon}}(\varepsilon,\bar{\varepsilon})$ as Z axis , the surface plot was showed in figure 4.

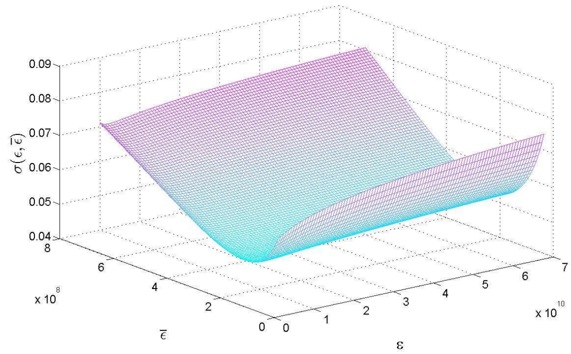

Fig.4. The surface plot of the root mean square error of the smoothed values, based on different coefficients of smoothing

Figure 4 shows that when $(\varepsilon,\bar{\varepsilon})$ equal to (9.3E+9, 1.9E+8) the $\sigma_{\varepsilon,\bar{\varepsilon}}(\varepsilon,\bar{\varepsilon})$ reaches the minimum value. So the coefficients of smoothing $(\varepsilon,\bar{\varepsilon})$ are 9.3E+9 and1.9E+8.

IV. RESULTS

Based on the time scale combined algorithm, the combined time scale was calculated, which was denoted as TA_{joint}. In Figure 5, The blue curve is the time difference between cesium fountain and UTC(NTSC). The orange curve is the time difference between masers time scale and UTC(NTSC), The black curve is the time difference between combined time scale and UTC(NTSC).

In Figure 6, the first curve is the phase difference between cesium fountain and combined time scale. The second curve is the frequency difference between masers time scale and combined time scale.

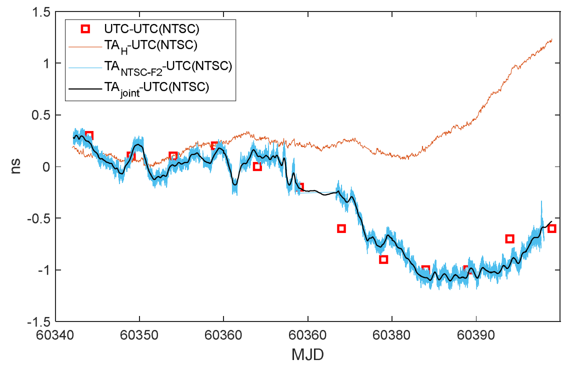

Fig.5. The time differences between TA_H, $TA_{NTSC\text{-}F2}$, TA_{joint} and UTC(NTSC)

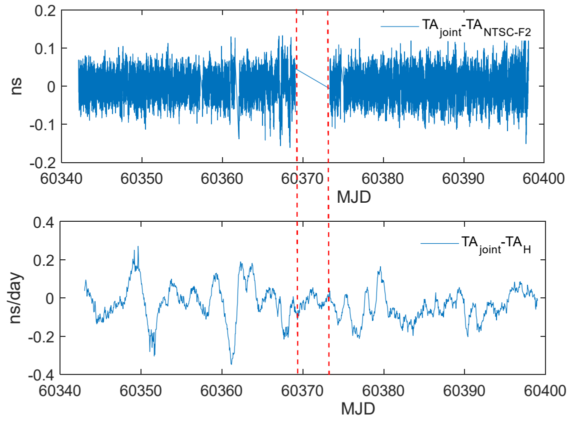

Fig.6. The differences between TA_H, $TA_{NTSC-F2}$ and TA_{joint}

In the above coefficients of smoothing selection, we hope that the combined time scale is closer to the cesium fountain, but when the cesium fountain NTSC-F2 is in dead time, it is closer to the masers time scale.

Figure 5 and figure 6 show that the deviation between TA_{joint} and $TA_{NTSC-F2}$ is smaller than the deviation between TA_{joint} and TA_H, indicating that TA_{joint} and $TA_{NTSC-F2}$ have better fitting. The deviation between TA_{joint} and TA_H is less than ±0.12ns/day when $TA_{NTSC-F2}$ is in the dead time, indicating that TA_H has a higher weight during this period.

Stability and accuracy are important indicators for evaluating time scales. First, the Allan deviation of the TA_{joint} ,$TA_{NTSC-F2}$ and TA_H was calculated separately, as shown in Figure 7. Secondly, the root mean square (RMS) of the TA_{joint},$TA_{NTSC-F2}$ and TA_H relative to UTC is calculated separately, as shown in Table 1.

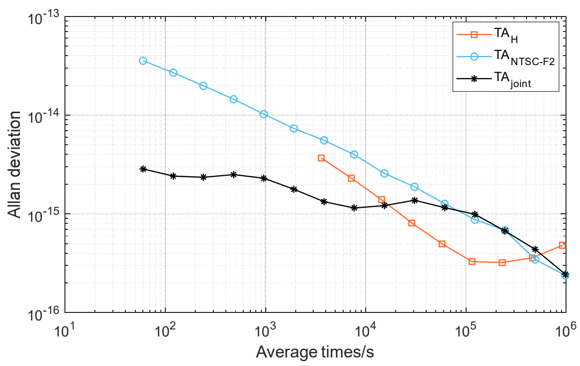

Fig.7. The frequency instability of TA_H , $TA_{NTSC-F2}$ and TA_{joint}

Figure 7. shows the short-term stability of the combined time scale is better than that of the cesium fountain. The combined time scale with Allan deviation 1.33E-15 at 3600s, is 76% better than that of cesium fountain. The long-term stability of the combined time scale is better than that of the masers time scale. The combined time scale with Allan deviation 2.45E-16 at 10 days, is 49% better than that of masers time scale.

TABLE I THE RMS OF THE TA_{JOINT}, $TA_{NTSC-F2}$ AND TA_H

	TAjoint -UTC	$TA_{NTSC-F2}$-UTC	TA_H-UTC
RMS/ns	0.14	0.15	0.97

Table 1. shows the RMS of the combined time scale is 0.14ns, 86% better than that of masers time scale, 6% better than that of cesium fountain.

Through the time scale combined algorithm, the short-term stability of the combined time scale is better than that of the cesium fountain, the long-term stability is better than that of the masers time scale, and the RMS is better than that of the cesium fountain and masers time scale. The performance of the time scale had been improved. So, this method can be effectively applied to combine the different atomic clocks or time scales. It uses the advantages of both series to improve the performance of the result. It provides an effective method for the performance improvement of the subsequent real-time physical signals.

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